

The correlation between the power-law coefficients in creep: the temperature dependence

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In this study we investigated the relationship between the coefficients of the Norton law for low-stress high-temperature deformation results of different materials reported in the literature. These coefficients are interrelated and it is shown that this relation can be theoretically deduced from the assumption of a variation of the activation area with the inverse of the effective stress (i.e. a variation of the activation energy with the logarithm of the stress). As a consequence, all the straight lines $\ln(\text{strain-rate})-\ln(\text{stress})$ intersected at the same point, named the pivot point. This pivot point is characteristic of each material. Such a correlation can be found in the field of the mechanics of materials when a power-law relation holds. © 1998 Kluwer Academic Publishers

1. Introduction

The variation of the strain-rate, $\dot{\epsilon}$ with the applied load is usually described by engineers by a power-law called the Norton law

$$\dot{\epsilon} = A_{\text{NL}}(\sigma/\sigma_0)^n \quad (1)$$

in which σ_0 is a reference stress and n the power coefficient. The Norton law coefficient, A_{NL} , is generally analysed in terms of an Arrhenius equation of the form

$$A_{\text{NL}} = C_1 \exp - (Q/RT) \quad (2)$$

where C_1 is a constant for a given material, Q is the apparent activation energy for the mechanism involved in the deformation process (usually close to the activation energy for self diffusion), and R is the gas constant.

On the other hand, in the materials science literature, for polycrystals such as alloys and ceramics at relatively high stresses and high temperatures, the Dorn equation is preferred which may be written as

$$\dot{\epsilon} = A_{\text{D}}[(D_{\text{L}}Gb)/RT](\sigma/G)^n \quad (3a)$$

or

$$\dot{\epsilon} = A'_{\text{D}}[(D_{\text{L}}\sigma_0 b)/RT](\sigma/\sigma_0)^n \quad (3b)$$

where G is the shear modulus, b the length of the appropriate Burger's vector, T the absolute temperature, D_{L} the lattice diffusion coefficient, n the stress exponent, the value of which varies from about 4–7 (and sometimes up to 20) depending on the type of flow, the temperature and the material under study, and $A_{\text{D}}(A'_{\text{D}})$ is a dimensionless constant which has values from the order of unity to as large as 10^{16} . Thus, Equation 3 is essentially empirical and it is

difficult to ascribe a physical signification to $A_{\text{D}}(A'_{\text{D}})$ and n .

According to the works of Ashby and co-workers [1–3] the coefficients A_{D} and n are not independent of each other. An empirical correlation was established between them which can be written as

$$\log_{10} A_{\text{D}} = B_1(n - B_2) \quad (4)$$

where B_1 (~ 3) and B_2 (~ 2.7) are constants from a class of materials. Derby and Ashby [3] showed that this equation has a physical meaning and can be explained by introducing a threshold or a friction stress into creep law with a constant A near 1 and a stress exponent $n = 3$. This last analysis leads to the exponent coefficient depending on the applied stress, in contradiction to experimental data for pure metal and solid solution alloys [4].

The purpose of this study is to illustrate the role of the temperature in Equation 1 by analysing already published data, and to find some physical significance for the relation between the coefficients A_{NL} and n in the Norton law.

2. Analysis of the Norton law coefficients

In a recent work by Bhattacharya *et al.* [5], the engineering equation for high-temperature deformation (Equation 1) in the case of linear $\ln(\text{strain-rate})-\ln(\text{stress})$ relationship was analysed in the form

$$\ln \dot{\epsilon} = \ln A_1 + n \ln \sigma \quad (5)$$

with

$$\ln A_1 = \ln C_1 - n \ln \sigma_0 - (Q/RT) \quad (6)$$

for 27 materials including glass-ceramics, ceramics, metals, alloys, superalloy, metal-matrix composites

and intermetallics. The literature data were analysed in terms of Equation 1 by means of the least squares method and the coefficients C_1 , Q , σ_0 and n are optimized to find the minimum in Equation 7 for a given temperature T_i

$$[\sum \ln A_{1i} - \ln C_1 + n_i \ln \sigma_0 + (Q/RT_i)]^2 = \text{minimum} \quad (7)$$

The best fits were obtained when the material constants $\ln C_1$, and $\ln \sigma_0$ are assumed to be independent of both stress and temperature, Q is near the activation energy as reported by previous authors (activation energy at a constant stress or Dorn analysis using a compensated stress) and n is a linear function of the temperature

$$n = a + (b/T) \quad (8)$$

A best fit can be sometimes found by adding a term in (c/T^2) , but Equation 8 is reported to give an accurate estimation of the strain-rate [5].

The strain-rates were calculated later with the computed values by Equations 5–8 and a very good agreement could be achieved with the experimental data (the error prediction is generally less than 2%) over the temperature and stress ranges considered. All the coefficients calculated by Bhattacharya *et al.* [5] are listed in Table I.

With the results reported in Table I, we calculated the relation between the coefficients n and C_1 in the form

$$n = \alpha + \beta \ln A_1 \quad (9)$$

A perfect correlation was found for all materials (the regression coefficients is $r^2 = 1$) where α and β are independent of the temperature and the stress ranges investigated. Equation 9 implies that, for a given material, all the straight lines $\ln \dot{\epsilon}$ versus $\ln T \sigma$ cross over to the same pivot point, $\dot{\epsilon}_p$, σ_p such as

$$\dot{\epsilon} = \dot{\epsilon}_p (\sigma/\sigma_p)^n \quad (10)$$

The coefficients α and β are easily calculated from Equations 5, 6, and 8

$$a + b/T = \alpha + \beta [\ln C_1 - (Q/RT) - a \ln \sigma_0 - (b/T) \ln \sigma_0] \quad (11)$$

and as α and β are temperature independent

$$\alpha = a(1 + \beta \ln \sigma_0) - \beta \ln C_1 \quad (12a)$$

$$\beta = -1/[(Q/bR) + \ln \sigma_0] \quad (12b)$$

From Equations 10 and 12 the coordinates of the pivot point become

$$\sigma_p = \exp(-1/\beta) = \sigma_0 \exp(Q/bR) \quad (13a)$$

$$\dot{\epsilon}_p = \exp(-\alpha/\beta) = \dot{\epsilon}_0 (\sigma_p/\sigma_0)^a \quad (13b)$$

The values we obtained for α , β , $\dot{\epsilon}_p$ and σ_p are reported in Table I and plotted in Figs 1 and 2. For example, for Si_3N_4 it is shown in Fig. 3 that when the deformation rate is plotted versus the applied stress in log–log coordinates, the straight lines intersected in the pivot point. The fit is remarkable with the points calculated from Equations 7 and 8 and with the experimental data.

TABLE I Q , C , σ_0 , a and b from [5] and calculated values of coefficients in Equations 9 and 10

Material	Q (KJ mol ⁻¹)	$\ln C_1$	$\ln \sigma_0$	a	b	α	β	$\ln \sigma_p$	$\ln \dot{\epsilon}_p$
9608-Glass	631	49.94	10.690	-0.0987	1 581.34	0.77	-0.0170	58.685	45.203
0329-Glass	800	65.23	6.910	-0.3400	1 910.84	0.84	-0.0175	57.266	48.109
3YTZP	468	22.88	2.150	0.7700	2 065.18	1.49	-0.0340	29.407	43.868
4YTZP	412	28.33	7.520	1.5212	434.32	1.66	-0.0082	121.618	201.896
52YTZT	502	24.93	2.760	11.1174	-12 123.00	13.71	0.4503	-2.221	-30.441
6YTZP	498	21.67	1.900	1.0221	617.90	1.22	-0.0101	98.840	120.754
12CETZP	491	22.96	2.250	1.8553	-914.28	1.55	0.0160	-62.344	-96.880
TZP20A	577	28.52	1.880	3.3469	-2 161.59	2.61	0.0331	-30.226	-78.936
3YTZP40A	680	39.45	3.730	0.2600	3 184.32	1.57	-0.0340	29.415	46.128
3YTZP60A	737	43.95	4.410	5.1343	-4 576.20	3.71	0.0668	-14.961	-55.506
3YTZP80A	749	44.38	3.350	2.0831	-339.25	1.94	0.0038	-262.203	-508.782
42YTZP20A	713	32.18	-0.850	-4.0790	11 047.71	0.08	-0.1447	6.913	0.516
BaTiO ₃	1072	77.67	2.440	-13.7737	23 903.94	0.43	-0.1276	7.834	3.374
Spinel	429	20.78	4.810	-5.0180	12 925.89	0.09	-0.1136	8.802	0.748
MA754	611	55.03	4.230	15.7841	-9 949.64	19.50	0.3168	-3.156	-61.555
IN100	644	3.81	3.810	-5.7693	10 988.30	-3.39	-0.0921	10.859	-36.859
Cu	167	12.72	3.410	0.4858	3 202.75	1.63	-0.1033	9.682	15.766
Mg	163	18.53	2.030	4.7595	506.76	4.98	-0.0246	40.718	202.667
W	611	14.23	1.940	4.6571	4 254.67	4.93	-0.0521	19.213	94.671
Comp-s10	196	34.33	2.620	-8.3617	15 152.84	5.11	-0.2395	4.176	21.321
Comp-s14	284	37.06	2.120	-8.1161	15 443.41	4.41	-0.2308	4.332	19.108
Comp-s7	102	22.67	2.960	-22.9530	23 437.04	3.06	-0.2871	3.483	10.655
Comp-s4	203	36.87	2.400	-12.4240	17 208.55	5.04	-0.2619	3.819	19.242
WC-Co	454	34.04	6.680	5.6824	-4 837.62	6.53	0.2170	-4.608	-30.102
Ni ₃ Si	556	45.35	3.520	-17.7576	26 636.38	0.13	-0.1658	6.031	0.767
Al-Cu	132	14.26	2.870	-1.7542	2 674.09	0.44	-0.1135	8.807	3.845
Pb-Sn	32	7.69	4.100	-0.3214	917.43	0.76	-0.1205	8.295	6.342
Pb-Sn	68	7.69	4.100	-0.3210	917.43	0.37	-0.0768	13.015	4.828

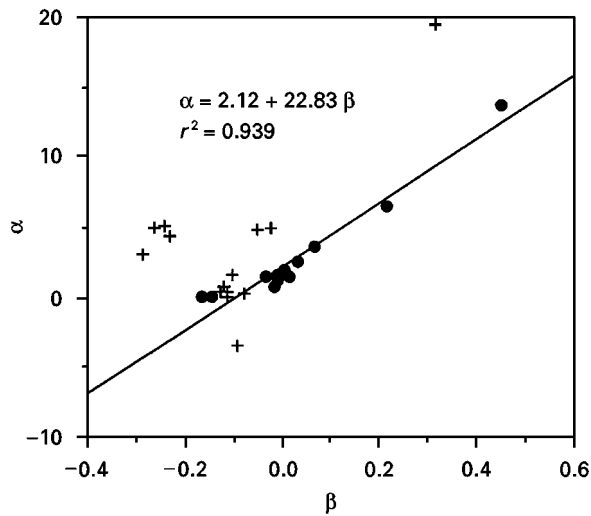


Figure 1 Variation of coefficients α and β in Equation 9 for the 27 materials investigated by Bhattacharya *et al.* [5]. Coefficients correlated for (●) ceramics and (+) other materials.

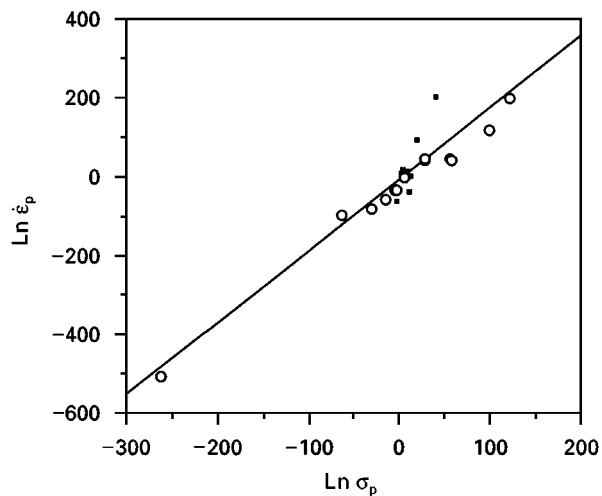


Figure 2 Variation of coefficients $\ln \dot{\epsilon}_p$ versus $\ln \sigma_p$ in Equation 10. (○) Coefficients correlated for (○) ceramics and (■) other materials.

The first observation which can be made is that the pivot point is not unique, but depends on the material investigated. A second observation is that the coefficients α and β are interrelated for the same class of materials. For example, for ceramics and glass-ceramics (Fig. 1)

$$\alpha = 2.12 + 22.8 \beta \quad (14)$$

This relation means that for the same class of materials, the pivot points lie in the same straight line in log-log coordinates (Fig. 2).

The third observation to be noticed is that the pivot point implies that the coefficients Q and σ_0 of Equation 6 are related by Equation 15

$$\exp - (Q/RT) \sigma_0^{-b/RT} = \exp - [(Q + b \ln \sigma_0)/RT] \quad (15)$$

A change in Q corresponds to a variation in σ_0 . The calculation scheme is thus indeterminate and it is therefore impossible to obtain Q and σ_0 independently.

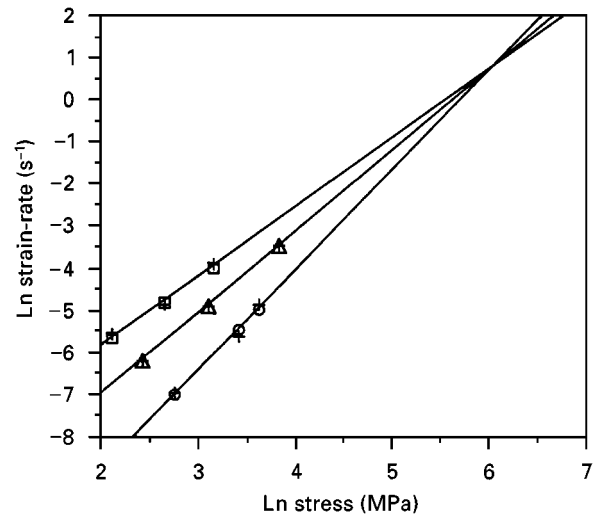


Figure 3 Variation of the natural logarithm of the strain-rate versus the natural logarithm of the applied stress for Ni-S₄ samples. (+) Experimental data, others symbols: calculated values from [5] with Equations 6–8. The straight lines intersect at the same point: the pivot point. (○) 1323 K, (△) 1353 K, (□) 1373 K.

TABLE II Variation of the apparent activation energy in Equation 6 with σ_0

σ_0 (MPa)	IN 100 Q (J mole ⁻¹)	MA 754 Q (J mole ⁻¹)	4 YTZP Q (J mole ⁻¹)	42 YTZP 20 A Q (J mole ⁻¹)
1	992 069	261 089	439 154	703 609
10	781 712	451 562	430 839	678 171
100	571 356	642 034	422 525	652 732
1000	360 999	832 507	414 210	627 295

For example, Table II gives different values of σ_0 and Q for IN100, MA74, 4YTZP and 42YTZP20A. It is obvious that for the two first materials the values of Q depend strongly on σ_0 , but the coordinates of the pivot point remain unchanged.

3. The physical meaning of the n -ln A relationship

In the preceding section we presented results obtained for the 27 materials investigated and showed that the pivot point is a consequence of the mathematical equations chosen to describe the Norton law coefficients in creep experiments. The existence of one empirical formula for a variety of materials supports the interpretation of the pivot point as something of fundamental significance. Our purpose is now to find a physical interpretation of Equation 1 when glide kinetics are described by an Arrhenius equation such as Equation 2 and a temperature dependence of the power-law exponent such as Equation 8.

It is now well known that plastic deformation of polycrystalline materials is thermally activated and that the strain-rate generally obeys to the Arrhenius relationship (Equation 2). The stress dependence of the activation energy, ΔG , is the activation area, A , with physical explanation corresponds to the area swept out by a dislocation segment in the activation

process

$$A = - (1/b) (\partial \Delta G / \partial \sigma^*)_T \quad (16a)$$

$$\sigma^* = \sigma - \sigma_i \quad (16b)$$

where σ^* is the effective (or thermally activated) component of the applied stress, and σ_i is the long-range internal stress or athermal component of the stress.

The activation area is a strong function of stress and is only weakly dependent on composition and grain size. If A is proportional to the inverse of the effective stress, as is commonly reported in the literature [6, 7]

$$A = B/\sigma^* \quad (17)$$

and by integrating Equation 16 with Equation 17, and provided that no structural alterations occur during the test and the mobile dislocation density remains constant, the activation energy becomes dependent on the logarithm of the effective stress, as shown for example by Anglada and Guiu [8]

$$\begin{aligned} \Delta G &= \int (\partial \Delta G / \partial \sigma^*)_T \partial \sigma^* = \int (Bb/\sigma^*) \partial \sigma^* \\ &= \Delta G_0 - Bb \ln(\sigma^*/\sigma_r^*) \end{aligned} \quad (18)$$

where σ_r^* is some arbitrary constant with units of stress introduced to avoid the singularity at $\sigma^* = 0$, and ΔG_0 the activation energy at $\sigma^* = 0$.

Substituting the dependence of the activation energy with the stress from Equation 18 into the rate equation leads to

$$\dot{\epsilon} = \dot{\epsilon}_0 (\sigma^*/\sigma_r)^{m^*} \exp - (\Delta G_0/RT) \quad (19a)$$

with

$$m^* = Bb/RT \quad (19b)$$

As, in general, the pre-exponential factor is proportional to the a th power of the applied stress [9], the strain-rate can be shown to be

$$\dot{\epsilon} = A' (\sigma^*/\sigma_r)^{a+Bb/RT} \exp - (\Delta G_0/RT) \quad (20)$$

This expression is the same as the Norton equation with the only difference being that the effective stress must be considered in place of the total applied stress.

The temperature dependence of the power-law coefficient, n , is not surprising, and the constancy of the product m^*T (equivalent to assuming a zero entropy of activation) has been observed by numerous searchers [10–12].

From these theoretical considerations it is shown that an inverse dependence of the surface activation versus the effective shear stress implies that the activation energy depends logarithmically on stress

$$\Delta G = \Delta G_0 \ln(\sigma_0^*/\sigma^*) \quad (21)$$

and the power-law representation for the creep process is a consequence of Equation 21. Such dependence of activation energy versus the logarithm of the stress was reported in the literature. For example, Seeger [13, 14] proposed a rate-controlling mechanism governed by the Peierls–Nabarro stress. This mechanism involved the formation of a pair of kinks

in the dislocation line corresponding to the activation enthalpy

$$H = H_K [1 + (1/4) \log(16 \sigma_{\text{peierls}}^0 / \pi \sigma^*)] \quad (22)$$

where H_K is the energy for the formation of a single kink, and $\sigma_{\text{peierls}}^0$ the Peierls–Nabarro stress at 0 K.

By substituting Equation 22 in the rate Equation 2, we obtained

$$\dot{\epsilon} = \dot{\epsilon}_0 \exp(-H_K/RT) [(\sigma^*/(16/\pi) \sigma_{\text{peierls}}^0)]^{H_K/4eRT} \quad (23)$$

where e is the base of the system of natural logarithms.

With Equation 10 the coordinates of the pivot point become

$$\sigma_p = (16 \times 10^4 \pi) \sigma_{\text{peierls}}^0 \quad (24a)$$

$$\dot{\epsilon}_p = \dot{\epsilon}_0 \quad (24b)$$

This result must be related to recent works by Wang and Nieh [15] where it is shown that the Peierls stress plays an important role in the power-law creep. Using extensive data for metals, ceramics, and silicates, it was observed that σ_0 in Equation 3b may be Peierls' stress and that A'_D is a function of this stress. These results are in good agreement with the coordinates obtained for the pivot point.

The variation of the activation energy with the logarithm of the effective stress is observed in other rate-controlling mechanisms, as in the Hartley [16] dislocation intersection dislocation mechanism and the Schoeck and Seeger [17] thermally activated cross-slip mechanism. This activation energy is reported to be expected if the stress exerted by a barrier on one element of the mobile dislocation varies inversely with the distance [16, 18]. It should also be noticed that other dependences of the activation energy on the logarithm of the stress have been reported by Li [19] and by Yaroszewicz and Ryvkina [20, 21] and leads directly to an inverse dependence of the activation area with the effective stress (Equation 17) and a power-law relationship between the effective stress and strain-rate (Equation 1) for the deformation rate process. In a recent work, Reed-Hill and Kaufman [18] reported that the power-law relationship holds if:

- (i) the internal stress is small enough to be neglected so $\sigma = \sigma^*$;
- (ii) dynamic strain ageing did not occur;
- (iii) the temperature is not high enough to allow dynamic recovery or dynamic recrystallization to alter the metallurgical structure.

With the first assumption, when the long-range internal stress is small enough to be neglected, the Norton law may apply and the pivot point, obtained from the results of Bhattacharya *et al.* [5], can be physically justified. On the other hand, a direct relation between the activation area and the inverse of the applied stress was found by Balasubramanian and Li [6] and leads directly to a power-law according to Equation 1.

4. Discussion and concluding remarks

The correlation (Equation 9) between the constant, A_1 , and the exponent, n , and the derived relationships

(Equation 10) is not limited for the plastic flow and the creep process, and was reported by numerous authors when mechanical properties are related by means of a power-law expression such as

$$Y = CX^n \quad (25)$$

Zilberstein [22] argued that the correlation exists well outside of mechanics of materials when a power-law holds, and such a correlation was related in many papers in the fields of the material science.

1. For the hardness test, in the microhardness range (typically when the applied load, P , varies between 5 and 500 gf), P is related to the indentation print, d , by means of the Meyer's law

$$P = Ad^n \quad (26)$$

When n is less than 2 (values of the exponent are often reported to vary between 1 and 2) the measured hardness of the material is dependent on the applied load and this phenomenon is known as the indentation size effect (ISE). It was reported in recent works by Li and Bradt on rutile, cassiterite [23] and lanthanum hexaboride [24] single crystals, that the coefficients A and n for Knoop microhardness are correlated (see also [25] for Brinell and Vickers hardness). It can be seen in Fig. 4 that the variation of n versus A is linear and the straight lines intersect in the same point.

It should be noticed that Meyer's law is only valid in a small interval of the test load and does not possess any physical significance. A relation between the apparent hardness and the reciprocal length of the indentation print is more appropriate for representing the ISE [26].

2. For chromium alloyed rail steel, Ghonem and Kalousek [27] showed on biaxial loading specimens, that the ratchetting rate varies with the parallel-to-flow shear stress as a function of the maximum compressive stress value, σ_{\max} , in a power-law $(\dot{\epsilon}_N)_s = k\tau_{\max}^n$. The coefficients k and n depend on σ_{\max} and are correlated (Fig. 5).

3. The coefficients in the Manson–Coffin law

$$N^n \Delta \epsilon_p = C \quad (27)$$

for creep resistance steels, Cr–Ni steels and their welded joints, are interrelated [25, 28].

But the main works are related to the fatigue crack growth rate and the relations between the Paris–Erdogan's coefficients

$$da/dN = C\Delta K^n \quad (28)$$

Numerous authors have point out the relation between m and C , such as Equations 9 and 10 [29–33], and Tanaka [34] called the point where all the straight lines da/dN versus ΔK intersected the “pivot point” for stage II fatigue crack growth rate. One relation was found for all the steels and another one for all the aluminium alloys [31]. In fact, it is difficult to understand why a unique point is found for one class of materials, so the physical meaning of such a relation was uncertain and some criticisms were made by McCartney and Irving [35] (the relation is a consequence of the dimensional equation), and by Cortie and Garret [36, 37] (the relation is an artefact

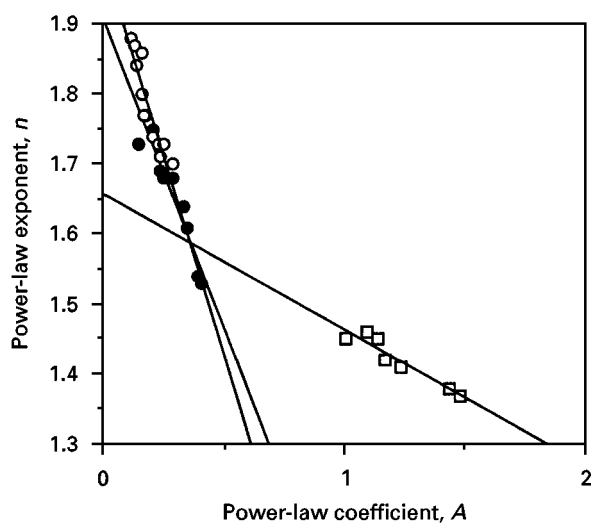


Figure 4 Variation of the power-law exponent in the Meyer's relationship with the power-law coefficient in Equation 26 for single crystals of (●) TiO_2 $n = 2.00 - 1.14 A$; (○) SnO_2 $n = 1.91 - 0.89 A$; and (□) LaB_6 $n = 1.66 - 0.19 A$, from Li and Bradt [23, 24].

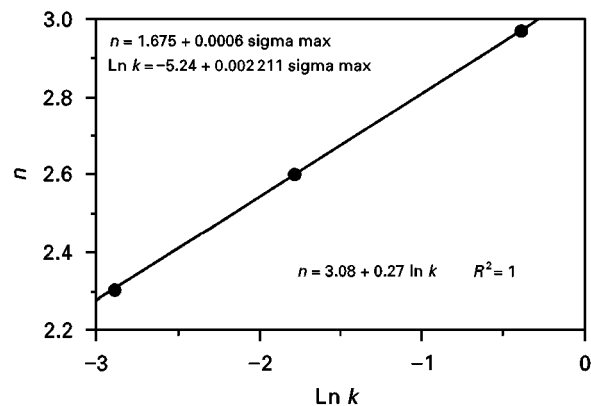


Figure 5 Correlation between the coefficients n and k of the variation of the ratchetting rate versus the maximum shear stress (from Ghonem and Kalousek's experimental results [27]).

produced by the logarithmic representation and is of little physical significance). These criticisms were refuted by Iost and Lesage [38] and it was shown that the earlier works were generally erroneous, because the pivot point depends on the material investigated.

The temperature dependence of the coefficients C and m were investigated by Iost [39] for steels, aluminium alloys, HS steels and superalloy. The coefficient m is temperature dependent and the Equations 9 and 10 hold again. For Jeglic *et al.*'s [40] works on Al–2.6 Mg alloy, the intersection between the representation of the variation of the fatigue crack growth rate versus the stress intensity factor range was observed in normal coordinates and the objection of Cortie and Garett [36, 37] was rejected. It was also found that Yokobori's kinetic theory of fatigue crack propagation [30], based on dislocation dynamics, gives a theoretical justification of the pivot point. This justification is based on an activation energy which depends on the logarithm of the stress and must be related to the above development to find a physical meaning to the n – $\ln A_1$ relationship to the Norton law.

In summary, this study has shown that the two constants in the Norton law for the flow process are not independent for a given material, with a constant structure, when the exponent, n , varies directly as the absolute temperature. Experimental verification of the linearity between n and $1/T$ has been observed in widely different materials [41] and then a correlation exists between the coefficients α and β when the temperature is varying. The reduction in the number of constants used to describe the deformation process may be useful for engineering characterization and can significantly improve the predictability in practical situations when no testing can be performed [42].

This correlation implies that all the straight lines representing the variation of the strain rate versus the applied load in logarithmic coordinates intersected at the same point, named the pivot point.

The pivot points depend on the material, but is interrelated for the same class of materials.

These relations are verified with a very good accuracy for the 27 materials reported by Bhattacharya *et al.* [5] and are physically related to a logarithmic variation of the activation energy with the applied stress, which corresponds to the inverse variation between the activation area and the stress (applied or effective) as reported earlier in a large number of experimental studies.

Such a correlation is not unique, but was reported earlier in the literature when the mechanical properties are described by a power law.

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